

Constraining Supersymmetric SO(10) Models Through Cosmology

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Abstract

We study the impact of the symmetry breaking patterns from supersymmetric SO(10) down to the standard model on the standard big-bang cosmology through the formation of topological defects. None of the models is consistent with the standard cosmology without invoking any mechanism to solve the monopole problem. For this purpose, we use a hybrid false vacuum inflationary scenario. Only two symmetry breaking patterns are consistent with these topological considerations and with the actual data on the proton lifetime.

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I. INTRODUCTION

Grand Unified Theories (GUTs) have been constructed to unify the strong, weak and electromagnetic interactions. The minimal grand unified group in which all kinds of matter are unified is $SO(10)$ GUT [1]. Indeed $SO(10)$ has a 16 dimensional spinorial representation and therefore all quarks and leptons belonging to a single family can be assigned to a single multiplet. Now, when looking at the measured values at LEP of the three gauge coupling constants and interpolating them to high energies, we find that they do not merge. On the other hand, the three coupling constants in the Minimal Supersymmetric Standard Model, with supersymmetry broken at $T \sim 10^3$ GeV, merge in a single point at $T \sim 10^{16}$ GeV [2]. Supersymmetry can also solve the gauge hierarchy problem.

Supersymmetric $SO(10)$ is consistent with the measured values of $\sin^2 \theta_w$ and α_s and the unification of the three gauge coupling constants at $\sim 10^{16}$ GeV [2]. It also beautifully solves the question of fermions masses [3]. Furthermore it leads to a relation for $\tan \beta$, an unknown factor within the Minimal Supersymmetric Standard Model, giving $\tan \beta = m_t/m_b$ [4]. Natural doublet-triplet splitting can be achieved in supersymmetric $SO(10)$ via the Dimopoulos-Wilczek mechanism [5]. $SO(10)$ also contains an unbroken matter parity which lies in the centre of $SO(10)$. The latter can suppress rapid proton decay and provide a good cold dark matter candidate in the form of the lightest superparticle. Now, introducing a 126 and a $\overline{126}$ into the supersymmetric model, the see-saw mechanism can be implemented [6], thus providing a good hot dark matter candidate; the right-handed neutrino gets a super-heavy Majorana mass and the left-handed neutrino gets a very small mass. Supersymmetric $SO(10)$ can also explain the solar neutrino problem via the MSW mechanism [7]. Finally, it is a good candidate for baryogenesis [8].

Thus, supersymmetric $SO(10)$ is very attractive from a particle physics point of view and can also help to solve some cosmological problems. One would therefore like to be able to select one of the breaking patterns. Unfortunately, there is considerable freedom in doing so, and the only way out from a particle physics point of view would be from string

compactification.

However, any particle physics model is irrelevant if it does not satisfy cosmological considerations. Conversely, any cosmological model is irrelevant if it does not agree with particle physics considerations. In other words, any GUT model is tied up with cosmology, and one should not be considered without the other; as nice as a GUT (respectively cosmological) model can be, it can however lead to a cosmological catastrophe (cannot be implemented in any viable particle physics model), and should therefore be regarded with suspicion. When symmetries spontaneously break down, according to Kibble mechanism [9], topological defects form, such as monopoles, strings or domain walls. Monopoles, because they would be too abundant, and domain walls, because they are too heavy, if present today would dominate the energy density of the universe and lead to a cosmological catastrophe. On the other hand, cosmic strings can explain structure formation and part of the baryon asymmetry of the universe.

We derive below the cosmological constraints on the symmetry breaking schemes of supersymmetric $SO(10)$ down to the standard model due to the formation of topological defects. In sec.II we list the possible symmetry breaking pattern involving at most one intermediate symmetry breaking scale. In sec. III, we review the conditions for the formation of topological defects, giving systematic conditions in supersymmetric $SO(10)$. In sec. IV we discuss the hybrid inflationary scenario which can be implemented in supersymmetric $SO(10)$. In sections V, VII and VIII we give a systematic analysis of the cosmological implications for the different symmetry breaking scenarios listed in section II. We conclude in section IX, pointing out the only models not in conflict with the standard cosmology.

II. BREAKING DOWN TO THE STANDARD MODEL

In this section, we give a list of all the symmetry breaking patterns from supersymmetric $SO(10)$ down to the standard model, using no more than one intermediate breaking scale. The main differences between supersymmetric and non-supersymmetric $SO(10)$ models is in

the symmetry breaking scales as we shall see and in the choice for the intermediate symmetry groups. In non-supersymmetric models, at least one intermediate symmetry breaking is needed in order to obtain consistency with the measured value of $\sin^2 \theta_w$ and with the gauge coupling constants interpolated to high energy to meet around 10^{15} GeV. On the other hand, in supersymmetric $SO(10)$ models, we can break directly down to the standard model, breaking supersymmetry at $\sim 10^3$ GeV, predicting the measured value of $\sin^2 \theta_w$ and having the gauge coupling constant joining in a single point at 2×10^{16} GeV.

We shall consider the following symmetry breaking patterns from supersymmetric $SO(10)$ down to the standard model:

1. $SO(10) \xrightarrow{M_{GUT}} SU(5) \times U(1)_X \xrightarrow{M_G} SM$
2. $SO(10) \xrightarrow{M_{GUT}} SU(5) \xrightarrow{M_G} SM$
3. $SO(10) \xrightarrow{M_{GUT}} SU(5) \times \widetilde{U(1)} \xrightarrow{M_G} SM$
4. $SO(10) \xrightarrow{M_{GUT}} SU(4)_c \times SU(2)_L \times SU(2)_R \xrightarrow{M_G} SM$
5. $SO(10) \xrightarrow{M_{GUT}} SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \xrightarrow{M_G} SM$
6. $SO(10) \xrightarrow{M_{GUT}} SU(3)_c \times SU(2)_L \times U(1)_R \times U(1)_{B-L} \xrightarrow{M_G} SM$
7. $SO(10) \xrightarrow{M_{GUT}} SM$
8. $SO(10) \xrightarrow{M_{GUT}} SU(5) \times U(1)_X \xrightarrow{M_G} SM \times Z_2$
9. $SO(10) \xrightarrow{M_{GUT}} SU(5) \times Z_2$ (1)
10. $SO(10) \xrightarrow{M_{GUT}} SU(5) \times \widetilde{U(1)} \xrightarrow{M_G} SM \times Z_2$
11. $SO(10) \xrightarrow{M_{GUT}} SU(4)_c \times SU(2)_L \times SU(2)_R \xrightarrow{M_G} SM \times Z_2$
12. $SO(10) \xrightarrow{M_{GUT}} SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \xrightarrow{M_G} SM \times Z_2$
13. $SO(10) \xrightarrow{M_{GUT}} SU(3)_c \times SU(2)_L \times U(1)_R \times U(1)_{B-L} \xrightarrow{M_G} SM \times Z_2$
14. $SO(10) \xrightarrow{M_{GUT}} SM \times Z_2$ (2)

where SM stands for the standard model gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$. In models

1. to **6.**, we break SUSY at $\sim 10^3$ GeV, and the symmetry group $SU(3)_c \times SU(2)_L \times U(1)_Y$ down to $SU(3)_c \times U(1)_Q$ at $\sim M_Z$. In models **7.** to **11.**, we also break SUSY at $\sim 10^3$ GeV, and we break the group symmetry $SU(3)_c \times SU(2)_L \times U(1)_Y \times Z_2$ down to $SU(3)_c \times U(1)_Q \times Z_2$ at $\sim M_Z$. In the latter cases, the Z_2 symmetry remains unbroken down to low energy, and acts as matter parity. It preserves large values for the proton lifetime and stabilizes the Lightest SuperParticle (LSP), thus providing a good hot dark matter candidate.

In order to satisfy LEP data, we must have $M_{GUT} \sim M_G$ (see Langacker and Luo in Ref. [2]). For non supersymmetric models, the value of the $B-L$ symmetry breaking scale is anywhere between 10^{10} to $10^{13.5}$ GeV [10]. For the supersymmetric case it is around 10^{15} to 10^{16} GeV. Indeed, the scale M_G is fixed by the unification of the gauge couplings, and in the absence of particle threshold corrections is $M_G \sim 10^{16}$ GeV [2]. But, as in the non-supersymmetric case, threshold corrections can induce uncertainties of a factor $10^{\pm 1}$ GeV. These corrections vary with the intermediate subgroup considered, but in any cases, we can assume that $M_G \sim 10^{15} - 10^{16}$ GeV. The scale M_{GUT} must be greater than the unified scale M_G and below the Planck scale, therefore we must have $10^{19} GeV \geq M_{GUT} \geq 10^{15} - 10^{16} GeV$.

In order to simplify the notation, we shall use the following

$$\begin{aligned}
a. \quad & 4_c 2_L 2_R \equiv SU(4)_c \times SU(2)_L \times SU(2)_R \\
b. \quad & 3_c 2_L 2_R 1_{B-L} \equiv SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \\
c. \quad & 3_c 2_L 1_R 1_{B-L} \equiv SU(3)_c \times SU(2)_L \times U(1)_R \times U(1)_{B-L} \\
d. \quad & 3_c 2_L 1_Y(Z_2) \equiv SU(3)_c \times SU(2)_L \times U(1)_Y (\times Z_2) \\
e. \quad & 3_c 1_Q(Z_2) \equiv SU(3)_c \times U(1)_Q (\times Z_2)
\end{aligned} \tag{3}$$

III. TOPOLOGICAL DEFECT FORMATION IN SUPERSYMMETRIC MODELS

In this section, we review the conditions for topological defect formation during phase transitions in the early universe associated with the spontaneous symmetry breaking of a

group G down to a subgroup H of G , showing first that the results derived in the non-supersymmetric case [9] are not affected by the presence of supersymmetry. We then apply the results to spontaneous symmetry breaking patterns from supersymmetric $SO(10)$. In a separate section, we study the formation of hybrid defects, such as monopoles connected by strings or domain walls bounded by strings, particularly looking at their cosmological impact [18,17].

A. Defects formation in supersymmetric models

We study here the conditions for defect formation in supersymmetric models. We show that the conditions for topological defect formation in non supersymmetric theories [9], are not affected by the presence of supersymmetry. We review these conditions with special application to supersymmetric $SO(10)$.

In non supersymmetric theories, the conditions for topological defect formation during the spontaneous symmetry breaking of a non-supersymmetric Lie group G to a non-supersymmetric Lie group H are well known; they are associated with the connection of the vacuum manifold $\frac{G}{H}$ [9]. Now one may worry about the non Lie nature of the superalgebra. Fortunately, it has been shown [11] that the superalgebra is Lie admissible and that the infinitesimal transformations of the superalgebra can be exponentiated to obtain a Lie superalgebra. The Lie admissible algebra is an algebraic covering of the Lie algebra, and it was first identified by Albert [12]. It is such a covering that allows a Lie admissible infinitesimal behavior while preserving the global structure of the Lie group. The graded Lie algebra is Lie admissible and therefore much of the Lie algebra theory may be extended to it with the appropriate modification. In particular, a connected (super)Lie group structure persists [13]. Hence, the formation of topological defects in supersymmetric models will be the same as in non-supersymmetric ones. Whether or not supersymmetry is broken at the phase transition will not affect the conditions under which topological defects form.

The defect formation and stability conditions are therefore as follows [9]. Consider the

spontaneous symmetry breaking of a group G down to a subgroup H of G . Topological defects, arising according to the Kibble mechanism [9] when G breaks down to H , are classified in terms of the homotopy groups of the vacuum manifold $\frac{G}{H}$ [9]. If the fundamental homotopy group $\pi_0(\frac{G}{H}) \neq I$ is non trivial, domain walls form when G breaks down to H . If the first homotopy group $\pi_1(\frac{G}{H}) \neq I$ is non trivial, topological cosmic strings form. If the second homotopy group $\pi_2(\frac{G}{H}) \neq I$ is non trivial, monopoles form. Note that when we denote a group G (respectively H), we really mean the supersymmetric version of this group, and when we write $SO(10)$ we mean its universal covering group $Spin(10)$ (supersymmetric) which is simply connected. If the group H breaks later to a subgroup K of H , we have the following conditions for the stability of the defects formed when G broke to H . If the fundamental homotopy group $\pi_0(\frac{G}{K})$ is non trivial, the walls are topologically stable, $\pi_1(\frac{G}{K})$ is non trivial, the strings are topologically stable and if $\pi_2(\frac{G}{K})$ is non trivial, the monopoles are topologically stable down to K . Domain walls, because they are too heavy, and monopoles, because they are too abundant according to the Kibble mechanism if present today, would dominate the energy density of the universe. Hence these defects are in conflict with the standard cosmology. On the other hand, cosmic strings can explain large scale structure, anisotropies in the Cosmic Background Radiation and part of the baryon asymmetry of the universe.

Now consider the phase transition associated with the breaking of $SO(10)$ down to a subgroup G of $SO(10)$, and apply the above results to this particular case. Since $Spin(10)$ is connected we have $\pi_2(\frac{SO(10)}{G}) = \pi_1(G)$ and $\pi_1(\frac{SO(10)}{G}) = \pi_0(G)$ and therefore the formation of monopoles and strings during the Grand Unified phase transition is governed by the non triviality of $\pi_1(G)$ and $\pi_0(G)$ respectively. If G breaks down later to a subgroup K of G , monopoles formed during the first phase transition will remain topologically stable after the second phase transition if $\pi_2(\frac{SO(10)}{K}) \neq I$. Strings formed during the first phase transition will be topologically stable after the next phase transition if $\pi_1(\frac{SO(10)}{K}) \neq I$.

B. Hybrid defects

When we have an intermediate breaking scale, we can also get mixed defects. There are two kinds of mixed defects that we can get in supersymmetric SO(10) models; they are monopoles connected by strings and domain walls bounded by strings. Their cosmological evolutions have been studied in a non supersymmetric general case [18,17].

1. Monopoles connected by strings

In supersymmetric SO(10) models, we can have monopoles connected by strings [17]. If the first phase transition leaves an unbroken U(1) symmetry which later breaks to unity, that is if the breaking pattern proceeds as

$$G \rightarrow H \times U(1)_x \rightarrow H \tag{4}$$

where G and H are both simply connected, then monopoles form at the first phase transition, and then get connected by strings at the following one. Indeed, the second homotopy group $\pi_2(\frac{G}{H \times U(1)}) = \pi_1(H \times U(1)) = Z$ indicates the formation of monopoles during the first phase transition in (4). These monopoles carry a $U(1)_x$ magnetic charge, and are topologically unstable. Now the first homotopy group $\pi_1(\frac{H \times U(1)}{H})$ is also non trivial, hence cosmic strings form at the second stage of symmetry breaking in (4). The strings connect monopole/antimonopole pairs of the first phase transition [17]. Because the whole system of strings rapidly decays [17], monopoles connected by strings do not seem to affect the standard cosmology in any essential way. On the other hand, if the universe undergoes a period of inflation between the two phase transitions, or if the phase transition leading to the formation of monopoles is itself inflationary, then the picture is very different. The decay of the system of strings is negligible. If the monopoles are inflated beyond the horizon, the strings form according to the Kibble mechanism and their evolution is that of topologically stable cosmic strings [17]. In this class of scenarios, with inflation and cosmic strings,

temperature fluctuations in the CBR measured by COBE give constraints on the scale of the phase transition leading to the string formation and on the scalar coupling constant [23].

2. Walls bounded by strings

The other kind of topological mixed defect that we can get in $SO(10)$ models are domain walls connected by strings. A first phase transition leaves an unbroken discrete symmetry, and cosmic strings form. At a subsequent phase transition, this discrete symmetry breaks leading to the formations of domain walls. They are bounded by the strings previously formed. Specifically, consider a symmetry breaking pattern of the form

$$G \rightarrow H \times Z_2 \rightarrow H \quad (5)$$

where G and H are both simply connected. The first homotopy group $\pi_1(\frac{G}{H \times Z_2}) = \pi_0(H \times Z_2) = Z_2$ thus Z_2 -strings form during the first phase transition in (5); they are topologically unstable. The discrete Z_2 symmetry breaking leads to the formation of domain walls at the second stage of symmetry breaking bounded by strings of the first phase transition. Such extended objects have been first studied by Kibble et al. [18]. They have shown that, in the non supersymmetric case, the cosmological relevance of these mixed objects depends on whether inflation occurs between the time when strings form and the time when the symmetry breaking leading to the formation of these walls occurs. The presence of supersymmetry does not affect the above conclusions. Following ref. [18], we get the following results. If the transition leading to the formation of the walls takes place without supercooling, the walls lose their energy by friction and disappear in a time $t_d \sim (t_W t_*)^{\frac{1}{2}}$ where t_W is the cosmic time corresponding to the the scale T_W at which the walls form and $t_* = \frac{3\alpha_G \eta_0}{32\pi\eta_3} \frac{M_p^2}{M_G^3}$, where η_3 is the effective massless degrees of freedom reflected by the walls and η_0 is the effective number of degrees of freedom in the supersymmetric $3_c 2_L 1_Y(Z_2)$ phase. With $\eta_3 = 33.75$ and $\eta_0 = 228.75$ we find $t_d \sim 10^{-33} - 10^{-36}$ sec for $T_W \sim 10^{15} - 10^{16}$ GeV and the corresponding scale $T_* \sim 10^9 - 10^{12}$ GeV. Therefore these extended objects do

not seem to affect the standard cosmology in any essential way. But if there is a period of inflation between the two phase transitions, the strings can be pushed to arbitrarily large scales; the walls form according to the Kibble mechanism and their evolution is that of topologically stable walls. The only difference from topologically stable Z_2 -walls is that the walls can now decay by the quantum nucleation of holes bounded by strings. Hole nucleation however is a tunneling process and is typically suppressed by a large exponential factor. The corresponding decay time is much larger than the time at which the walls come to dominate the universe, thereby upsetting standard cosmology.

IV. INFLATION IN SUPERSYMMETRIC $SO(10)$ MODELS

Since $SO(10)$ is simply connected and the standard model gauge group involves an unbroken $U(1)$ symmetry which remains unbroken down to low energy, all symmetry breaking patterns from supersymmetric $SO(10)$ down to the standard model automatically involve the formation of topologically stable monopoles. Even if some monopoles are connected by strings, a large fraction of them will remain stable down to low energy. Hence some mechanism has to be invoked in order to obtain consistency with the standard cosmology, such as an inflationary scenario. In this section, we discuss a false vacuum hybrid inflationary scenario which is the most natural mechanism for inflation in global supersymmetric $SO(10)$ models [23]. The superpotential in the inflaton sector is similar to that studied in [16]. We can note first that $SO(10)$ is rank 5, whereas the standard model gauge group $3_c 2_L 1_Y$ is 4. Hence the rank of the group has to be lowered from one unit at some stage of the symmetry breaking. This can be done using a pair of $16 + \overline{16}$ dimensional Higgs representation, or a pair of $126 + \overline{126}$ dimensional ones if the Z_2 parity is to be kept unbroken, as in models 8. to 14.. We can use a scalar field singlet under $SO(10)$ in order to force this pair of Higgs to get their VEV on the order of the GUT scale. The superpotential in this sector will be of the form

$$\alpha \mathcal{S} \overline{\Phi} \Phi - \mu^2 \mathcal{S} \tag{6}$$

where $\Phi + \overline{\Phi}$ stand for a pair of $16 + \overline{16}$ dimensional Higgs representations or a pair of $126 + \overline{126}$ dimensional Higgs representations, and $\frac{\mu}{\sqrt{\alpha}}$ is assumed to be the Grand Unified breaking scale. We then identify the scalar field \mathcal{S} with the inflaton field.

The evolution of the fields is as follows (a complete discussion of the potential in a general supersymmetric case is studied in ref. [16] and in a specific supersymmetric $SO(10)$ model is studied in reference [23]). The fields take random initial values, just subject to the constraint that the energy density is at the Planck scale. The inflaton field is distinguished from the other fields from the fact that the gradient of the GUT potential with respect to the inflaton field is very small. Therefore the non inflaton fields, except the Φ and $\overline{\Phi}$ fields, will roll very quickly down to their minimum at an approximately fixed value for the inflaton. Inflation occurs as the inflaton rolls slowly down the potential. The symmetry breaking implemented with the $\Phi + \overline{\Phi}$ fields occurs at the end of inflation and associated topological defects are not inflated away [16,23].

V. $SU(5)$ AS INTERMEDIATE SCALE

We shall describe in this section the symmetry breaking patterns from supersymmetric $SO(10)$ involving an $SU(5)$ intermediate symmetry. When the intermediate scale involves $SU(5)$ as a subgroup, say cases **1**, **2** and **7**, the scale M_G has to be $\sim 10^{16}$ GeV, and consequently the scale M_{GUT} is pushed close to the string compactification scale. $SO(10)$ can break via $SU(5)$ in four different ways. It can break via $SU(5) \times U(1)_X$, $SU(5)$, via $SU(5) \times \widetilde{U(1)}$ and via $SU(5) \times Z_2$, which correspond to models **1** and **8**, **2**, **3** and **10** and **9** respectively.

A. Breaking via $SU(5) \times U(1)_X$

We consider here two symmetry breaking patterns,

$$SO(10) \xrightarrow{M_{GUT}} SU(5) \times U(1)_X \tag{7}$$

$$\xrightarrow{M_G} SU(3)_c \times SU(2)_L \times U(1)_Y (\times Z_2) \quad (8)$$

$$\xrightarrow{M_Z} SU(3)_c \times U(1)_Q (\times Z_2) \quad (9)$$

with and without the Z_2 symmetry unbroken down to low energy. The latter is necessary to preserve large values for the proton lifetime and to stabilize the LSP. It can arise only if a pair of $126 + \overline{126}$ dimensional Higgs representations are used to lower the rank of the group, and hence must be part of the standard model gauge group in order to give large Majorana mass to the right-handed neutrino.

The $U(1)_X$ commutes with $SU(5)$. The X and Y directions are orthogonal to each other, and thus the $U(1)_X$ symmetry breaks down to unity at M_G (or to Z_2 if a pair of $126 + \overline{126}$ Higgs fields are used to break $SU(5) \times U(1)_X$). This feature is going to affect the formation of topological defects.

The first homotopy group $\pi_1(SU(5) \times U(1)_X) = Z$ is non trivial and thus topological monopoles form when $SO(10)$ breaks. They have a mass $M_m \geq 5 \times 10^{17}$ GeV. At the following phase transition the $U(1)_X$ symmetry breaks to unity (to Z_2) and hence cosmic strings (Z_2 -strings) form. They connect monopole-antimonopole pairs previously formed (see section III B 1). They have a mass per unit length $\sim 10^{32} \text{ GeV}^2$.

When $SU(5) \times U(1)_X$ breaks down to $3_c 2_L 1_Y (Z_2)$ new lighter monopoles form. Indeed, since $U(1)_X$ breaks down to unity (to Z_2) we consider the second homotopy group $\pi_2(\frac{SU(5)}{3_c 2_L 1_Y})$ to look for monopoles formations at M_G . Hence topologically stable monopoles form. They have a mass $M_m \sim 10^{17}$ GeV. They are topologically stable. Their topological charge may change from Y to Q.

Since monopoles form at both phase transitions and since the lighter ones are topologically stable, the inflationary scenario, as in section IV, is unable to solve the monopole problem. Hence these two models are inconsistent with observations.

B. Breaking via $SU(5)$

Here, $SO(10)$ breaks down to the standard model with intermediate $SU(5)$ symmetry alone. In this case, there is no interest in going to a larger Grand Unified group. The breaking scheme is

$$SO(10) \xrightarrow{M_{GUT}} SU(5) \quad (10)$$

$$\xrightarrow{M_G} SU(3)_c \times SU(2)_L \times U(1)_Y \quad (11)$$

$$\xrightarrow{M_Z} SU(3)_c \times U(1)_Q \quad (12)$$

which is that of model **1**. Since $SO(10)$ and $SU(5)$ are both simply connected, no topological defects form during the first stage of symmetry breaking.

The second homotopy group $\pi_2(\frac{SU(5)}{3_{c2L1Y}}) = Z$ hence topological monopoles form when $SU(5)$ breaks down to the standard model. The monopoles carry Y topological charge. The second homotopy group $\pi_2(\frac{SU(5)}{3_{c1Q}}) = Z$ which shows that the monopoles are topologically stable. They have a mass $M_m \sim 10^{17}$ GeV. Their topological charge may change from Y to Q.

Since the rank of $SO(10)$ is 5 and the rank of $SU(5)$ is 4, if we use an inflationary scenario as described in sec. IV to solve the monopole problem, the inflaton field will couple to a pair of $16 + \overline{16}$ Higgs fields representations which will be used to break $SO(10)$. The monopoles described above will form at the end of inflation, and their density will be high enough to dominate the universe. Hence this model is in conflict with the standard cosmology. It is also inconsistent with the actual data on the proton lifetime.

C. Breaking via $SU(5) \times \widetilde{U(1)}$

More interesting is the breaking via flipped $SU(5)$

$$SO(10) \xrightarrow{M_{GUT}} SU(5) \times \widetilde{U(1)} \quad (13)$$

$$\xrightarrow{M_G} SU(3)_c \times SU(2)_L \times U(1)_Y \quad (14)$$

$$\xrightarrow{M_Z} SU(3)_c \times U(1)_Q \quad (15)$$

Note that with flipped $SU(5)$, rather than using $SO(10)$ for the Grand Unified gauge group, the monopole problem is avoided [19]. The $\widetilde{U(1)}$ contains part of the electromagnetic gauge group $U(1)_Q$. The above symmetry breaking can only be implemented in supergravity $SO(10)$ models [19].

The first homotopy group $\pi_1(SU(5) \times \widetilde{U(1)}) = Z$ and therefore the first phase transition leads to the formation of topological monopoles when $SO(10)$ breaks. Furthermore, since $\pi_1(3_c 2_L 1_Y) = \pi_1(3_c 1_Q) = Z$ and $\widetilde{U(1)}$ contains part of the $U(1)_Y$ and $U(1)_Q$ symmetries, these monopoles are topologically stable. They have a mass $M_m \geq 5 \times 10^{17}$ GeV. They carry $B-L$, and their topological charge may change to Y and then to Q . Embedded cosmic strings form after the second stage of symmetry breaking [26].

We should be able to cure the monopole problem with an hybrid inflationary scenario for supergravity models. Indeed, since the rank of $SU(5) \times \widetilde{U(1)}$ is 5, the inflaton field can couple to the Higgs needed to break $SU(5) \times \widetilde{U(1)}$, and embedded strings will form at the end of inflation. Hence from a defects point of view the model is interesting, but appears to be inconsistent with the actual data for proton lifetime [27] and does not provide any Majorana mass for the right-handed neutrino. The latter problems are solved if we break $SU(5) \times \widetilde{U(1)}$ down to $3_c 2_L 1_Y Z_2$. In that case, a $126 + \overline{126}$ dimensional Higgs representation is used to break $SU(5) \times \widetilde{U(1)}$. Since the first homotopy groups $\pi_1(\frac{SU(5) \times \widetilde{U(1)}}{3_c 2_L 1_Y Z_2}) = Z_2$ and $\pi_1(\frac{SU(5) \times U(1)}{3_c 1_Q Z_2}) = Z_2$, topologically stable Z_2 -strings also form. They have a mass per unit length $\sim 10^{32}$ GeV².

D. Breaking via $SU(5) \times Z_2$

We consider here the breaking of $SO(10)$ via $SU(5)$ with added parity. The symmetry breaking is

$$SO(10) \xrightarrow{M_{GUT}} SU(5) \times Z_2 \quad (16)$$

$$\xrightarrow{M_G} SU(3)_c \times SU(2)_L \times U(1)_Y \times Z_2 \quad (17)$$

$$\xrightarrow{M_Z} SU(3)_c \times U(1)_Q \times Z_2 \quad (18)$$

where the unbroken Z_2 symmetry is a subgroup of the Z_4 centre of $SO(10)$. It plays the role of matter parity. It preserves large values for the proton lifetime and stabilizes the LSP, thus the model is consistent with the actual data on proton decay and provide a good hot dark matter candidate.

Now the fundamental homotopy group $\pi_0(SU(5) \times Z_2) = Z_2$ and therefore Z_2 cosmic strings form during the first phase transition. They have a mass per unit length $10^{38} GeV^2 \geq \mu \geq 10^{32} GeV^2$. Since the Z_2 symmetry is kept unbroken down to low energy, these strings remain topologically stable. They have been widely studied in the non supersymmetric case [25].

As in section VB, it is clear that topologically stable monopoles form during the second phase transition with mass $M_m \sim 10^{17} GeV$. Hence as in section VB, the model is in contradiction with observations.

We conclude that the only symmetry breaking pattern from $SO(10)$ down to the standard model with intermediate $SU(5)$ symmetry consistent with observations, is

$$SO(10) \rightarrow SU(5) \times \widetilde{U(1)} \rightarrow 3_c 2_L 1_Y Z_2 \rightarrow 3_c 1_Q Z_2 \quad (19)$$

where the Z_2 symmetry must be kept unbroken in order to preserve large values for the proton lifetime. The above symmetry breaking can only be implemented in supergravity models.

VI. PATTERNS WITH A LEFT-RIGHT INTERMEDIATE SCALE

In this section we study the symmetry breaking patterns from supersymmetric $SO(10)$ down to the standard model involving an $SU(2)_L \times SU(2)_R$ intermediate symmetry. These

are the symmetry breaking patterns with intermediate $4_c 2_L 2_R(Z_2)$ or $3_c 2_L 2_R 1_{B-L}$ symmetry groups. We show that these models, due the unbroken $SU(2)_L \times SU(2)_R$ symmetry share a property, which can make them cosmologically irrelevant, depending on the Higgs field chosen to implement the symmetry breaking. We then give a full study of the formation of the topological defects in each model.

A. Domain walls in left-right models

We study here a property shared by the symmetry breaking schemes from $SO(10)$ down to the standard model, with or without unbroken parity Z_2 ,

$$SO(10) \xrightarrow{M_{GUT}} G \xrightarrow{M_G} 3_c 2_L 1_Y(Z_2) \quad (20)$$

where G is either $4_c 2_L 2_R$ or $3_c 2_L 2_R 1_{B-L}$. In these models, the intermediate scale involves an unbroken $SU(2)_L \times SU(2)_R$ symmetry, and consequently the intermediate symmetry group can be invariant under the charge conjugation operator, depending on the Higgs multiplet chosen to break $SO(10)$. The latter leaves an unbroken discrete Z_2^C symmetry which breaks at the following phase transition. In this case, the general symmetry breaking scheme given in equation (20) should really be written as

$$SO(10) \xrightarrow{M_{GUT}} G \times Z_2^C \xrightarrow{M_G} SM(\times Z_2) . \quad (21)$$

If $G = 4_c 2_L 2_R$, the discrete Z_2^c symmetry appears if the Higgs used to break $SO(10)$ is a single 54 dimensional representation [20]. If $G = 3_c 2_L 2_R 1_{B-L}$ the Z_2^c symmetry appears if a single 210 dimensional Higgs representation is used, with appropriate parameter range in the Higgs potential [21]. The appearance of the discrete Z_2^c symmetry leads to a cosmological problem [18]. Indeed, since $Spin(10)$ is simply connected, $\pi_1(\frac{SO(10)}{G \times Z_2^c}) = \pi_0(G \times Z_2^c) = Z_2$ and therefore Z_2 strings form during the first phase transition associated with the breaking of $SO(10)$. They have a mass per unit length $\sim 10^{32} - 10^{34} \text{ GeV}^2$. When the discrete Z_2^C symmetry breaks, domain walls form bounded by the strings of the previous phase

transition. Some closed walls can also form. As shown in section III B, these domain walls do not affect the standard cosmology in any essential way. On the other hand, if a period of inflation occurs between the two phase transition, or if the phase transition leading to the walls formation is itself inflationary, then the evolution of the walls is that of topologically stable Z_2 walls. They dominate the universe, destroying the standard cosmology.

B. Breaking via $4_c 2_L 2_R$

We now consider the symmetry breaking of $SO(10)$ via the Pati-Salam gauge group $4_c 2_L 2_R$ subgroup of $SO(10)$ which later breaks down to the standard model gauge group with or without matter parity

$$SO(10) \xrightarrow{M_{GUT}} SU(4)_c \times SU(2)_L \times SU(2)_R \quad (22)$$

$$\xrightarrow{M_G} SU(3)_c \times SU(2)_L \times U(1)_Y (\times Z_2) \quad (23)$$

$$\xrightarrow{M_Z} SU(3)_c \times U(1)_Q (\times Z_2) \quad (24)$$

with supersymmetry broken at $\simeq 10^3$ GeV and the scales M_{GUT} and M_G respectively satisfy $M_{pl} \geq M_{GUT} \geq 10^{16}$ GeV and $M_G \sim 10^{15} - 10^{16}$ GeV. The discrete Z_2 symmetry is kept unbroken if we use a pair of $126 + \overline{126}$ -Higgs dimensional representation to break $4_c 2_L 2_R$, and is broken if we use a pair of $16 + \overline{16}$ dimensional Higgs. The unbroken Z_2 symmetry plays the role of matter parity, preserving large values for the proton lifetime and stabilizing the LSP. Hence only the model with unbroken Z_2 at low energy is consistent with the actual value for proton lifetime.

If a single 54 dimensional Higgs representation is used to break $SO(10)$, equation (24) should really be written as [18]

$$Spin(10) \xrightarrow{M_{GUT}} \left(\frac{Spin(6) \times Spin(4)}{Z_2} \right) \times Z_2^C \quad (25)$$

$$\xrightarrow{M_G} SU(3)_c \times SU(2)_L \times U(1)_Y (\times Z_2) \quad (26)$$

$$\xrightarrow{M_Z} SU(3)_c \times U(1)_Q (\times Z_2) \quad (27)$$

where we have explicitly shown the hidden symmetry. A Z_2 symmetry has to be factored out in equation (25) since $Spin(6)$ and $Spin(4)$ have a non trivial intersection. The overall Z_2^c is generated by the charge conjugation operator; it is unrelated to the previous Z_2 one. Subsequently, the Z_2^c discrete symmetry is broken. If a pair of Higgs in the $126 + \overline{126}$ representation are used to break $4_c 2_l 2_R$, then a new Z_2 symmetry emerges, as described above; it is unrelated to the previous ones. The standard model gauge group is broken with a Higgs in the 10 dimensional representation of $SO(10)$.

If a single 210-Higgs multiplet is used to break $4_c 2_l 2_R$, with appropriate range in the parameters of the Higgs potential, the Z_2^C does not appear [21].

1. Monopoles

The non trivial intersection of $Spin(6)$ and $Spin(4)$ leads to the production of superheavy monopoles [17] when $SO(10)$ breaks to $4_c 2_L 2_R$. These monopoles are superheavy with a mass $M_m \geq 10^{17}$ GeV. They are topologically unstable.

Since the second homotopy group $\pi_2(\frac{4_c 2_L 2_R}{3_c 2_L 1_Y(Z_2)}) = Z$ is non trivial, new monopoles form when $4_c 2_L 2_R$ breaks down to the standard model gauge group. They are unrelated to the previous monopoles. Furthermore, since the second homotopy group $\pi_2(\frac{4_c 2_L 2_R}{3_c 1_Q(Z_2)}) = Z$ is also non-trivial, these lighter monopoles are topologically stable. They have a mass $M_m \sim 10^{16} - 10^{17}$ GeV. These monopoles form according to the Kibble mechanism, and their density is such that, if present today, they would dominate the energy density of the universe.

2. Domain walls

If a 54 dimensional Higgs representation is used to break $SO(10)$ down to $4_c 2_L 2_R$, the symmetry breaking is given by equation (25) which is of the form of equation (21) with $G = 4_c 2_L 2_R$, so that a discrete Z_2^c symmetry emerges at the intermediate scale. Thus, as shown in section VI A, Z_2 -strings form during the first phase transition. (They are unrelated

to any of the monopole just discussed above.) During the second stage of symmetry breaking, this Z_2^C breaks leading to the formation of domain walls which connect the strings previously formed. These walls bounded by strings do not affect the standard cosmology in any essential way. But if there is a period of inflation before the phase transition leading to the walls formation takes place (see section III B), the walls would dominate the energy density of the universe, leading to a cosmological catastrophe.

3. Cosmic strings

Now we consider the models where $4_c 2_L 2_R$ breaks down to the standard model gauge group with added Z_2 parity, as in model 8. Then a new Z_2 symmetry emerges at M_G , which is unrelated to the previous ones. Since $\pi_1(\frac{4_c 2_L 2_R}{3_c 2_L 1_Y Z_2}) = Z_2$ Z_2 -strings form when $4_c 2_L 2_R$ breaks. They have a mass per unit length $\mu \sim 10^{30} - 10^{32} \text{ GeV}^2$. Since the Z_2 symmetry is then kept unbroken down to low energy, we break the standard model gauge group with a Higgs 10-plets. The strings are topologically stable down to low energy.

Density perturbations in the early universe and temperature fluctuations in the CBR generated by these strings could be computed.

4. Solving the monopole problem

In order to solve the monopole problem, we use an hybrid inflationary scenario, as discussed in section IV. The rank of both $4_c 2_L 2_R$ and $4_c 2_L 2_R Z_2$ is four. Therefore the inflaton field will couple to a pair of Higgs field which will break $4_c 2_L 2_R$. The primordial monopoles formed when $\text{SO}(10)$ breaks are diluted by the inflation. But then lighter monopoles form at the end of inflation when $4_c 2_L 2_R$ breaks, which are topologically stable. In the case of unbroken Z_2 parity cosmic strings also form. Monopole creation at this later stage make the model inconsistent with observations.

If $\text{SO}(10)$ is broken with a 54 dimensional Higgs representation, domain walls will form through the Kibble mechanism at the end of inflation, which will dominate the universe, as

shown in section VI A, hence leading to a cosmological catastrophe.

We conclude that the model is cosmologically inconsistent with observations. It is inconsistent whether or not the discrete Z_2^C symmetry is unbroken at the intermediate scale.

C. Breaking via $3_c 2_L 2_R 1_{B-L}$

We can break via $3_c 2_L 2_R 1_{B-L}$ and then down to the standard model with or without the discrete Z_2 symmetry preserved at low energy

$$SO(10) \xrightarrow{M_{GUT}} SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \quad (28)$$

$$\xrightarrow{M_G} SU(3)_c \times SU(2)_L \times U(1)_Y (\times Z_2) \quad (29)$$

$$\xrightarrow{M_Z} SU(3)_c \times U(1)_Q (\times Z_2) \quad (30)$$

The Z_2 symmetry, which can be kept unbroken down to low energy if only safe representations are used to implement the symmetry breaking, plays the role of matter parity. It preserves large values for the proton lifetime. Hence only models with unbroken Z_2 parity at low energy are consistent with the actual values of proton decay. If $SO(10)$ is broken with a single 210-Higgs multiplet, with the appropriate range of the parameters in the Higgs potential [21], then there appears a discrete Z_2^c symmetry at the intermediate scale which is generated by the charge conjugation operator, and the symmetry breaking really is

$$SO(10) \xrightarrow{M_{GUT}} SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times Z_2^c \quad (31)$$

$$\xrightarrow{M_G} SU(3)_c \times SU(2)_L \times U(1)_Y (\times Z_2) \quad (32)$$

$$\xrightarrow{M_Z} SU(3)_c \times U(1)_Q (\times Z_2) . \quad (33)$$

The Z_2^c is unrelated to the Z_2 symmetry which can be added to the standard model gauge group in equations (32) and (33). If one uses a combination of a 45 dimensional Higgs representation with a 54 dimensional one to break $SO(10)$, then the symmetry breaking is that of equation (28), and no discrete symmetry appears as in (31) [22]. The rest of the symmetry breaking is implemented with a pair of $16 + \overline{16}$ -Higgs multiplets or with a pair

of $126 + \overline{126}$ -Higgs multiplets if matter parity is preserved at low energy. $3_c 2_L 1_Y$ is broken with a 10-Higgs multiplet.

1. Monopoles

The first homotopy groups $\pi_1(3_c 2_L 2_R 1_{B-L}) = Z$, $\pi_1(3_c 2_L 1_Y) = Z$ and $\pi_1(3_c 1_Q) = Z$, showing that topologically stable monopoles are produced during the first phase transition from $SO(10)$ down to $3_c 2_L 2_R 1_{B-L}$. They have a mass $M_m \geq 10^{17}$ GeV. These monopoles are in conflict with cosmological observations.

2. Domain walls

If $SO(10)$ is broken with a single 210 dimensional Higgs representation then the symmetry breaking is that of equation (35). Hence, as in the breaking pattern (25), the appearance of the discrete Z_2^c symmetry leads to the formation of non-stable cosmic strings during the first symmetry breaking and to the formation of domain walls in the breaking of $3_c 2_L 2_R 1_{B-L}$ down to the standard model gauge group. The cosmological relevance of these walls bounded by strings depends upon the presence of an inflationary epoch before the phase transition leading to the walls formation has taken place, see sec. VI A.

3. Embedded Defects

In these models with intermediate $3_c 2_L 2_R 1_{B-L}$ symmetry, the breaking schemes are equivalent to

$$SO(10) \xrightarrow{M_{GUT}} G \times SU(2)_R \times U(1)_{B-L} \xrightarrow{M_G} G \times U(1)_Y (\times Z_2) \xrightarrow{M_Z} 3_c 1_Q (Z_2) \quad (34)$$

where $G = SU(3)_c \times SU(2)_L$. In direct analogy with electroweak strings [24], it is easy to see that embedded defects form during the second stage of symmetry breaking. They have a mass per unit length $\mu \sim 10^{30} - 10^{32} \text{ GeV}^2$. The stability conditions for these strings can be

computed. If these strings are dynamically stable, they may generate density perturbations in the early universe and temperature anisotropy in the microwave background.

4. Cosmic Strings

Consider the model where $3_c 2_L 2_R 1_{B-L}$ breaks down to $3_c 2_L 1_Y Z_2$. The first homotopy group $\pi_1(\frac{3_c 2_L 2_R 1_{B-L}}{3_c 2_L 1_Y Z_2}) = Z_2$ is non trivial which shows the formation of topological Z_2 strings. Since the Z_2 parity symmetry is kept unbroken down to low energy, the strings are topologically stable. They have a mass per unit length $\mu \sim 10^{30} - 10^{32} \text{ GeV}^2$. These strings will generate density perturbations in the early universe and temperature anisotropy in the microwave background.

5. Solving the monopole problem

One can use an inflationary scenario as described in section (IV) to dilute the monopoles formed at M_{GUT} . Since the rank of $3_c 2_L 2_R 1_{B-L}(Z_2^c)$ is four, the inflaton field will couple to a pair of $16 + \overline{16}$ or $126 + \overline{126}$ which will break $3_c 2_L 2_R 1_{B-L}$, (see section IV). Cosmic strings (if unbroken Z_2 symmetry at low energy) and/or domain walls (if unbroken Z_2^c symmetry at the intermediate scale) will form at the end of inflation. As shown in section VIA the presence of this inflationary epoch between the two phase transitions at M_{GUT} and M_G respectively would make the walls dominate the energy density of the universe, (see sec. VIA). Now the unbroken Z_2 symmetry is necessary to preserve large values for the proton life time, hence the only symmetry breaking pattern consistent with cosmology with intermediate $3_c 2_L 2_R 1_{B-L}$ symmetry is

$$SO(10) \xrightarrow{<45>+<54>} SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \quad (35)$$

$$\xrightarrow{<126>+<\overline{126}>} SU(3)_c \times SU(2)_L \times U(1)_Y \times Z_2 \quad (36)$$

$$\xrightarrow{<10>} SU(3)_c \times U(1)_Q \times Z_2 \quad (37)$$

where $SO(10)$ is broken with a combination of a 45 dimensional Higgs representation and 54 dimensional one, $3_c 2_L 2_R 1_{B-L}$ is broken with pair of $126 + \overline{126}$ dimensional Higgs representation and $3_c 2_L 2_Y Z_2$ is broken with a 10 Higgs multiplet.

VII. BREAKING VIA $3_c 2_L 1_R 1_{B-L}$

We shall consider first the symmetry breaking with intermediate $3_c 2_L 1_R 1_{B-L}$ without conserved matter parity at low energy

$$SO(10) \xrightarrow{M_{GUT}} SU(3)_c \times SU(2)_L \times U(1)_R \times U(1)_{B-L} \quad (38)$$

$$\xrightarrow{M_G} SU(3)_c \times SU(2)_L \times U(1)_Y \quad (39)$$

$$\xrightarrow{M_Z} SU(3)_c \times U(1)_Q \quad (40)$$

The first homotopy group $\pi_1(3_c 2_L 1_R 1_{B-L}) = Z + Z$ and therefore topological monopoles form during the first phase transition from supersymmetric $SO(10)$ down to $3_c 2_L 1_R 1_{B-L}$. These monopoles carry R and $B-L$, and have a mass $M_m \geq 10^{16} - 10^{17}$ GeV. Now $\pi_1(3_c 2_L 1_Y)$ and $\pi_1(3_c \times 1_Q)$ are both non trivial and hence, from an homotopy point of view, the monopoles are topologically stable. But as we are going to show below, some of these monopoles are indeed topologically stable, but some others will decay. During the second phase transition, the formation of strings is governed by the first homotopy group $\pi_1(\frac{3_c 2_L 1_R 1_{B-L}}{3_c 2_L 1_Y}) = Z$ showing the formation of cosmic strings during the second phase transition. These are associated with the breaking of $U(1)_R \times U(1)_{B-L}$ down to $U(1)_Y$ where the unbroken $U(1)_R \times U(1)_{B-L}$ symmetry in the first stage of symmetry breaking is responsible for the formation of monopoles. Now the weak hypercharge $\frac{Y}{2}$ is a linear combination of $B-L$ and R , $\frac{Y}{2} = (\frac{B-L}{2} + R)$. Therefore primordial monopoles with topological charge $\frac{B-L}{2} - R \neq 0$ get connected by the strings at the second stage of symmetry breaking. Some infinite and closed strings can also form. These cosmic strings are topologically unstable. They can break producing monopole-antimonopole pairs at the free ends. The monopole/antimonopole pairs connected by strings annihilate in less than a Hubble time and could produce the observed baryon asymmetry

of the universe [23]. Other monopoles formed during the first phase transition do not get connected by strings and remain stable down to low energy.

The monopole problem can be solved with an inflationary scenario as described in section IV. Since the rank of $3_c 2_L 1_R 1_{B-L}$ is 5, the inflaton field will couple to the Higgs mediating the second phase transition associated with the breaking of $3_c 2_L 1_R 1_{B-L}$. The monopoles can be pushed beyond the present horizon, and the monopole problem solved. Furthermore, since all the monopoles are inflated away, the string decay probability is negligible and the evolution of strings is identical to that of topologically stable strings. We therefore have a very interesting breaking scheme, where monopoles are created during a first transition, inflated away before cosmic strings which can explain galaxy formation, form.

This model where $3_c 2_L 1_R 1_{B-L}$ breaks down to the standard model without matter parity is in conflict with the actual data for proton lifetime. The solution to this problem is therefore that the intermediate subgroup break down to $3_c 2_L 1_Y Z_2$ as in model **13**. In this case, topologically stable Z_2 -strings will form during the second phase transition. They have a mass per unit length $\mu \sim 10^{30} - 10^{32} \text{ GeV}^2$. This interesting model with inflation and cosmic strings is studied in detail elsewhere [23].

VIII. BREAKING DIRECTLY TO THE STANDARD MODEL

Supersymmetric $SO(10)$ can break directly down to the standard model as in model **7**

$$SO(10) \xrightarrow{M_{GUT}} SU(3)_c \times SU(2)_L \times U(1)_Y \xrightarrow{M_Z} SU(3)_c \times U(1)_Q \quad (41)$$

or as in model **14**

$$SO(10) \xrightarrow{M_{GUT}} SU(3)_c \times SU(2)_L \times U(1)_Y \times Z_2 \xrightarrow{M_Z} SU(3)_c \times U(1)_Q \times Z_2 \quad (42)$$

with (42) or without (41) the Z_2 symmetry, subgroup of the Z_4 centre of $SO(10)$, unbroken down to low energy. The latter plays the role of matter parity, giving large values for the proton lifetime and stabilizing the LSP. The symmetry breaking occurs at $M_{GUT} \simeq$

$2 \times 10^{16} GeV$. The scenario without the unbroken Z_2 symmetry (41) is not, with the present data for proton decay, relevant phenomenologically. The Z_2 symmetry is also necessary for stabilizing the LSP and to provide a good cold dark matter candidate.

In model (42), the Z_2 symmetry remains unbroken down to low energy preserving large values for the proton lifetime. Furthermore, the first homotopy group $\pi_1(\frac{SO(10)}{3_c 2_L 1_Y Z_2}) = \pi_0(3_c 2_L 1_Y Z_2) = Z_2$ and therefore cosmic strings form when $SO(10)$ breaks. They are associated with the unbroken Z_2 symmetry and since the latter remains unbroken down to low energy, the strings are topologically stable down to low energy. They have a mass per unit length $\mu \sim 10^{32} GeV^2$. The latter could account for the density perturbations produced in the early universe which lead to galaxy formation and to temperature fluctuations in the CMBR.

Again, due to the unbroken $U(1)_Y$ symmetry, monopoles form at the Grand Unified phase transition. They carry Y topological charge and are topologically stable down to low energy. Their topological charge may change from Y to Q .

Since monopoles form in both models, the potential conflict with the standard big bang cosmology is again not avoided. Nevertheless, in model (11), if the Higgs field leading to monopole production takes its VEV before inflation ends and the latter ends before the Higgs leading to cosmic string formation acquires its VEV then we are left with a very attractive scenario.

Unfortunately, it does not seem possible to achieve this. If one attempts to inflate away the monopoles with a superpotential of the form given in section IV, an intermediate scale is introduced. Thus, one is either left with the monopole problem in cosmology or loses the simplicity of this breaking scheme.

IX. CONCLUSIONS

The aim of this paper is to constrain supersymmetric $SO(10)$ models which lead to the formation of topological defects through cosmological considerations. The main reason

for considering supersymmetric versions of the Grand Unified gauge group $SO(10)$ rather than non-supersymmetric ones, is to predict the measured values of $\sin^2 \theta_w$ and the gauge coupling constants merging in a single point at $\sim 2 \times 10^{16}$ GeV. Spontaneous Symmetry Breaking patterns from supersymmetric $SO(10)$ down to the standard model differ from non-supersymmetric ones firstly in the scale of $B-L$ symmetry breaking and secondly in the ways of breaking from $SO(10)$ down to the standard model. For non-supersymmetric models the scale of $B-L$ breaking has to be anywhere between 10^{10} and $10^{13.5}$ GeV whereas it is 10^{15} to 10^{16} GeV in supersymmetric models. Furthermore, in the supersymmetric case, we can break directly down to the standard model without any intermediate breaking scale, and not more than one intermediate scale is expected. We have given a systematic analysis of topological defects formation and their cosmological implications in each model. We found that the rules for topological defect formation are not affected by the presence of supersymmetry and since $SO(10)$ is simply connected and the standard model gauge group involves an unbroken $U(1)$ symmetry, all SSB patterns from supersymmetric $SO(10)$ down to the standard model involve automatically the formation of topologically stable monopoles. In tables 1, 2, 3 and 4 we give a summary of all the defects formed in each model. In the models where Z_2 -walls arise at the second phase transition, we have in fact hybrid defects. The walls are bounded by the Z_2 -strings previously formed and are unstable. In order to solve the monopole problem, we propose an hybrid inflationary scenario [15,16,23] which arise in supersymmetric $SO(10)$ models without imposing any external symmetry and without imposing any external field [23]. The inflationary scenario can cure the monopole problem, but then stabilizes the Z_2 walls previously discussed. Hence these cases lead to another cosmological problem. Imposing also that the models satisfy the actual data on the proton lifetime, we found that there are only two spontaneous symmetry breaking patterns consistent with cosmological considerations. Breaking directly to the standard model at first sight seems attractive. Unfortunately, one is unable to inflate away the monopoles without the introduction of an intermediate scale. The only breaking schemes consistent with cosmology correspond to the intermediate symmetry groups $3_C 2_L 2_R 1_{B-L}$, where $SO(10)$ is broken with a combination of

a 45 dimensional Higgs representation and a 54 dimensional one, and $3_C 2_L 1_R 1_{B-L}$. These intermediate symmetry groups must later break down to the standard model with unbroken matter parity; the symmetry breaking must be implemented with only Higgs fields in 'safe' representations [27], hence the rank of the group must be lowered with a pair of Higgs in the $126 + \overline{126}$ dimensional representation, and the standard model gauge group broken with a 10 dimensional one. The model with intermediate $3_C 2_L 1_R 1_{B-L}$, inflation and cosmic strings, is studied in detail elsewhere [23]. In supergravity SO(10) models, the breaking of SO(10) via flipped SU(5) is also possible.

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APPENDIX A:

G	$SO(10) \rightarrow G$	$G \rightarrow 3_c 2_L 1_Y$	cosmological problems
$SU(5) \times U(1)_X$	monopoles-1	monopoles-2 + strings	monopoles-2 + proton lifetime (Z_2 broken)
$SU(5)$	no defects	monopoles	monopoles + proton lifetime (Z_2 broken)
$SU(5) \times \widetilde{U(1)}$	monopoles	embedded strings	proton lifetime (Z_2 broken)
$4_c 2_L 2_R$	monopoles-1	monopoles-2	monopoles-2 + proton lifetime (Z_2 broken)
$4_c 2_L 2_R Z_2^c$	monopoles-1 + Z_2 -strings	monopoles-2 Z_2 -walls	Z_2 -walls and monopoles-2 + proton lifetime (Z_2 broken)
$3_c 2_L 2_R 1_{B-L}$	monopoles	embedded strings	proton lifetime (Z_2 broken)
$3_c 2_L 2_R 1_{B-L} Z_2^c$	monopoles + Z_2 -strings	embedded strings + Z_2 -walls	Z_2 -walls + proton lifetime (Z_2 broken)
$3_c 2_L 1_R 1_{B-L}$	monopoles	strings	proton lifetime (Z_2 broken)

Table 1 : This is a table showing the formation of topological defects in the possible symmetry breaking patterns from supersymmetric $SO(10)$ down to the standard model with broken matter parity. These models are inconsistent with proton lifetime measurements. The table also shows the relevant cosmological problems associated with each symmetry breaking pattern, when occurring within a hybrid inflationary scenario. From a topological defect point of view, models with intermediate $SU(5) \times \widetilde{U(1)}$, $3_c 2_L 2_R 1_{B-L}$ and $3_c 2_L 1_R 1_{B-L}$ symmetry groups are compatible with observations. The model with an intermediate $SU(5) \times \widetilde{U(1)}$ symmetry is only possible in supergravity $SO(10)$ models.

G	$SO(10) \rightarrow G$	$G \rightarrow 3_c 2_L 1_Y Z_2$	cosmological problems
$SU(5) \times U(1)_X$	monopoles-1	monopoles + Z_2 -strings	monopoles-2
$SU(5) \times Z_2$	Z_2 -strings	monopoles-2	monopoles-2
$SU(5) \times \widetilde{U(1)}$	monopoles	Z_2 -strings	no problem, monopoles inflated away
$4_c 2_L 2_R$	monopoles-1	monopoles-2 + Z_2 -strings	monopoles-2
$4_c 2_L 2_R Z_2^c$	monopoles-1 + Z_2 -strings	monopoles-2 + Z_2 -strings + Z_2 -walls	monopoles-2 + Z_2 -walls
$3_c 2_L 2_R 1_{B-L}$	monopoles	embedded strings + Z_2 -strings	no problem, monopoles inflated away
$3_c 2_L 2_R 1_{B-L} Z_2^c$	monopoles + Z_2 -strings	embedded strings + Z_2 -strings + Z_2 -walls	Z_2 -walls
$3_c 2_L 1_R 1_{B-L}$	monopoles	Z_2 -strings	no problem, monopoles inflated away

Table 2 : This is a table showing the formation of topological defects in the possible symmetry breaking patterns from supersymmetric $SO(10)$ down to the standard model with unbroken matter parity. These models are consistent with proton life time measurements and can provide a superheavy Majorana mass to the right-handed neutrinos. The table also shows the relevant cosmological problems associated with each symmetry breaking pattern, when occurring within a hybrid inflationary scenario. The models with intermediate $SU(5) \times \widetilde{U(1)}$, $3_c 2_L 2_R 1_{B-L}$ and $3_c 2_L 1_R 1_{B-L}$ symmetry groups are consistent with observations. The model with intermediate $SU(5) \times \widetilde{U(1)}$ symmetry is only possible in supergravity $SO(10)$ models.

$SO(10) \rightarrow 3_c 2_L 1_Y$	cosmological problems
monopoles-2	monopoles-2 + proton lifetime (Z_2 broken)

Table 3 : This is a table showing the formation of topological defects in models where supersymmetric $SO(10)$ breaks directly down to the MSSM with broken matter parity. The table also shows the relevant cosmological problems associated with the symmetry breaking pattern, when occurring within a hybrid inflationary scenario. These models are inconsistent with observations.

$SO(10) \rightarrow 3_c 2_L 1_Y Z_2$	cosmological problems
monopoles-2 + Z_2 -strings	monopoles-2

Table 4 : This is a table showing the formation of topological defects in models where supersymmetric $SO(10)$ breaks directly down to the MSSM with unbroken matter parity. The table also shows the relevant cosmological problems associated with the symmetry breaking pattern, when occurring within a hybrid inflationary scenario. These models are inconsistent with observations.

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